MATHEMATICS EXAMINATION
MARKING GUIDE 2nd TERM, 2023
GATSIBO DISTRICT
SENIOR THREE

SECTION A: Attempt all questions (55 marks)

1. $\frac{a+b\sqrt{2}}{c} = \frac{4+\sqrt{2}}{4-\sqrt{2}}$

$\frac{(4+\sqrt{2})(4+\sqrt{2})}{(4-\sqrt{2})(4+\sqrt{2})} = \frac{16+4\sqrt{2}+4\sqrt{2}+(\sqrt{2})^2}{16-(\sqrt{2})^2}$ (1 mark)

$= \frac{16+8\sqrt{2}+2}{16-2}$

$\frac{a+b\sqrt{2}}{c} = \frac{18+8\sqrt{2}}{14}$ (1 mark)

So, $a = 18$, $b = 8$, $c = 14$

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2. a) $100 \longrightarrow \frac{9\phi \times 6\phi}{1\phi} = 54$ marks
$1 \longrightarrow$
$60 \longrightarrow$
(1 mark) (1 mark)

Any method 1 mark
Answer = 1 mark

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3. b) $\%D = \frac{D \times 100}{H.P}$ where D: Discount
HP: Marked price
SP: Selling price

$D = M.P - SP = 43,000 Frw - 23,000 Frw = 20,000$ (1 mark)

$\%D = \frac{20,000 \times 100}{43,000} = \frac{2000}{43} = 46.5\%$ (1 mark)

3. Midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$\begin{cases} \frac{-4+x}{2} = -3 \Rightarrow -4+x=-6 \\ \qquad\qquad x=-6+4 \\ \qquad\qquad x=-2 \\ \frac{-5+y}{2} = 1 \Rightarrow -5+y=2 \\ \qquad\qquad y=2+5 \\ \qquad\qquad y=7 \end{cases}$$

(1 mark)

(1 mark)

$$\boxed{\begin{array}{l} x=-2 \\ y=7 \end{array}}$$

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4. $\frac{x+1}{x+2} \times \frac{x+3}{x-4}$

$\Rightarrow (x+1)(x-4) = (x+3)(x+2)$ (1 mark)

$\Rightarrow x(x-4)+1(x-4) = x(x+2)+3(x+2)$

$\Rightarrow x^2 - 4x + x - 4 = x^2 + 2x + 3x + 6$

$\Rightarrow -3x - 4 = 5x + 6$ (1 mark)

$-3x - 5x = 6 + 4$

$-8x = 10$

$x = \frac{10}{-8}$ (1 mark)

$x = -\frac{10}{8} \Rightarrow x = -\frac{5}{4}$

$S = \{-5/4\}$

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5. a) $81^{x-3} \times 27^{2-x} = \frac{1}{243^x}$

$\Rightarrow 3^{4(x-3)} \cdot 3^{3(2-x)} = \frac{1}{3^{5x}}$

$\Rightarrow 3^{4x-12+6x-3x} = 3^{-5x}$ (1 mark)

$\Rightarrow 3^{9x-12+6} = 3^{-5x}$

$\Rightarrow 3^{x-6} = 3^{-5x}$ (1 mark)

$x-6 = -5x$

$x+5x = 6$

$6x = 6$

$x = 1$ (1 mark)

b) $134_n = 54_8$

$(1\times n^2) + (3\times n^1) + (4\times n^0) = (5\times 8^1) + (4\times 8^0)$ (1 mark)

$n^2 + 3n + 4 = 40 + 4$

$n^2 + 3n - 40 = 0$ (1 mark)

$\Delta = b^2 - 4ac$

$= 3^2 - 4.1.40$

$= 9 + 160$

$\sqrt{\Delta} = \sqrt{169}$

$= \pm 13$

$n_1 = \frac{-3+13}{2.1} = \frac{10}{2} = 5$ (1 mark)

$n_2 = \frac{-3-13}{2.1} = -\frac{16}{2} = -8$ rejected

$\underline{n = 5}$

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6. $\frac{1}{2}(x-2) \geq 6 + x$

$\frac{1}{2}x - \frac{1}{2}\cdot 2 \geq 6 + x$

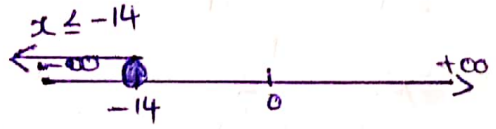
$\frac{1}{2}x \geq 6 + x + 1$

$\frac{1}{2}x - x \geq 7$

$x - 2x \geq 7.2$

$-x \geq 14$

$x \leq -14$ (1 mark).



(1 mark)

$S =]-\infty, -14]$

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7. a) $\frac{12m^2 - 27}{2m+3} = \frac{3(4m^2 - 9)}{2m+3} = \frac{3[(2m-3)(2m+3)]}{(2m+3)} = \underline{6m - 9}$

(1 mark) (1 mark)

b) N° of sides $= \frac{360°}{2x} = \frac{360}{24} = \underline{15 \text{ sides}}$

(1 mark) (1 mark)

8. a) $\vec{a} + \vec{b} - \vec{c} = \begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}1\\-4\end{pmatrix} - \begin{pmatrix}5\\2\end{pmatrix}$ (1 mark)

$= \begin{pmatrix}2+1-5\\3-4-2\end{pmatrix} = \begin{pmatrix}-2\\-3\end{pmatrix}$ (1 mark)

$\|a+b-c\| = \sqrt{(-2)^2+(-3)^2} = \sqrt{4+9}$

$\|a+b+c\| = \sqrt{13}$ (1 mark)

b) $x\vec{a} + y\vec{b} = \vec{c}$

$x\begin{pmatrix}2\\3\end{pmatrix} + y\begin{pmatrix}1\\-4\end{pmatrix} = \begin{pmatrix}5\\2\end{pmatrix}$ (1 mark)

$\begin{cases} 2x + y = 5 \\ 3x - 4y = 2 \end{cases} \quad \begin{vmatrix}3\\-2\end{vmatrix}$

$6x + 3y = 15$
$-6x + 8y = -4$
$11y = 11$
$y = 1$ (1 mark)

$2x + 1 = 5$
$2x = 5 - 1$
$2x = 4$
$x = 2$ (1 mark)

$S = \{(x,y); (2,1)\}$

9. $x+1 + 4x-7 + 2x+1 = 30$
$7x = 30+5$ (1 mark)
$7x = 35$
$x = 5$ (1 mark)

total ratio $2+3+4 = 9$ (1/2 mark)
$\sum$ of angles of triangle $= 180^\circ$
$\angle A = \frac{180 \times 2}{9} = 40^\circ$ (1/2 marks)
$\angle B = \frac{180 \times 3}{9} = 60^\circ$ (1/2 marks)
$\angle C = \frac{180 \times 4}{9} = 80^\circ$ (1/2 marks)

10. a) $f(x) = x^2 - 4$ and $g(x) = \sqrt{x}$

(i) $fg(x) = (\sqrt{x})^2 - 4$ (1 mark)

$= x - 4$

$fg(4) = 4 - 4$ (1 mark)

$fg(4) = 0$ (1 mark)

or

$g(4) = \sqrt{4}$

$= 2$

$fg(4) = 2^2 - 4$

$= 4 - 4$

$= 0$

(ii) $f(x) = x^2 - 4$

So, $x^2 - 4 = y$ (1 mark)

$\sqrt{x^2} = \sqrt{y+4}$

$x = \sqrt{y+4}$

$f(x)^{-1} = \sqrt{x+4}$ (1 mark)

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11. a) total $= 12 + 8 = 20$ pens (1 mark)

$P_r = \frac{12}{20} = 0.6$ (1 mark)

b) Son x

man $x + 24$

After 2 years $\Rightarrow$ Son $= x + 2$

man $= x + 24 + 2$

EQUATION $= x + 26 = 2(x + 2)$ (1 mark)

$x + 26 = 2x + 4$

$x - 2x = 4 - 26$

$-x = -22$

$x = 22$ (1 mark)

Son $= 22$

man $= 22 + 24 = 46$

12. a) $k = yx^2 = 1.25 \cdot 2^2$ (1 mark)

$= 1.25 \cdot 4$

$k = 5.$ (1 mark)

$y = \frac{k}{x^2} = \frac{5}{(\frac{1}{4})^2} = \frac{5}{\frac{1}{16}} = \frac{5 \times 16}{1} = 80$

$\underline{\underline{y = 80}}$ (1 mark)

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13. $A = P(1+r)^t$ $5\% = 0.05$

$= 200{,}000(1+0.05)^2$ (1 mark) (Method 2 ⇒ Answer 1)

$= 200{,}000(1.05)^2$ (1 mark)

$\underline{\underline{A = 220{,}500 \text{ FRW}}}$ (1 mark)

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14. Suppose that $t = 2010$ is $t = 0$

$t = 2018$ is $t = 8$

Coordinates (x, y) $(\underset{x_1}{0}, \underset{y_1}{2000})$ and $(\underset{x_2}{8}, \underset{y_2}{2600})$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2600 - 2000}{8 - 0} = \frac{600}{8} = 75$ (1 mark)

$y - y_1 = m(x - x_1)$

$y - 2000 = 75(x - 0)$

$\underline{\underline{y = 75x + 2000}}$ (1 mark)

in 2023 $y = (75 \times 13) + 2000$, where $2023 - 2010 = 13$ years.

$= \underline{\underline{2.975}}$ population (1 mark)

15. a) $\frac{4x^2}{4} = \frac{64}{4}$

$x^2 = 16$ (1 mark)

$x^2 - 16 = 0$

$(x-4)(x+4) = 0$

$\begin{cases} x-4=0 \\ x+4=0 \end{cases} \parallel \begin{matrix} x=4 \\ x=-4 \end{matrix}$ (1 mark)

$S = \{-4, 4\}$ (Any method = 1, Answer = 1)

b) $2^n = 128$ (1 mark)

$2^n = 2^7$

$n = 7$ (1 mark)

SECTION B:

16. a) $\begin{cases} x + y = 5 & (I) \\ x^2 + y^2 = 13 & (II) \end{cases}$

$x = 5 - y$ (I) (1 mark)

Substitute in equation (II)

$(5-y)^2 + y^2 = 13$ (1 mark)

$25 + 10y + y^2 + y^2 = 13$

$2y^2 + 10y + 25 - 13 = 0$

$2y^2 + 10y + 12 = 0$ (1 mark)

$2(y^2 + 5y + 6) = 0$

$\Delta = 25 - 6.4.1$

$= 25 - 24$

$= 1$

$\sqrt{\Delta} = \sqrt{1} = \pm 1$

$y_1 = \frac{5+1}{2.1} = \frac{6}{2} = 3$ (1 mark)

$y_2 = \frac{5-1}{2.1} = \frac{4}{2} = 2$ (1 mark)

so, $x = 5 - 3$

$= 2$

or $x = 5 - 2$

$= 3$

$\Rightarrow$ $x = 2, y = 3$ or

$y = 2, x = 3$

16.b) $3x^3 - 2x^2 - 12x + 8 = 0$

	3	−2	−12	8
(1 mark) −2	↓	−6	16	−8
	3	−8	4	0

(1 mark) (1 mark) (1 mark) (1 mark)

$(x+2)(3x^2 - 8x + 4) = 0$ (1 mark)

$\Delta = 64 - 4.4.3$ (1 mark)

$= 64 - 48$

$\sqrt{\Delta} = \sqrt{16}$

$= \pm 4$ (1 mark)

$x_1 = \frac{8+4}{2.3} = \frac{12}{6} = 2$ (1 mark)

$x_2 = \frac{8-4}{2.3} = \frac{4}{6} = \frac{2}{3}$ (1 mark)

$S = \{-2, 2/3, 2\}$

17. $y = (x+1)^2 \Rightarrow x^2 + 2x + 1$

Do not write in this margin

a) $x = \frac{-b}{2a} = \frac{-2}{2.1} = \frac{-2}{2} = -1$ (1 mark)

$f(-1) = (-1)^2 + 2(-1) + 1$

$= 1 - 2 + 1$

$= 0$

Vetex $(-1, 0)$ (1 mark)

b) ~~x~~ - intercept, when $x = 0$

$f(0) = 0^2 + 2.0 + 1$

$= 0 + 0 + 1$

$= 1$ $(0, 1)$ (1 mark)

x- intercept when $y = 0$

$x^2 + 2x + 1 = 0$ (1 mark)

$(x+1)^2 = 0$ $(-1, 0)$ (1 mark)

$x = -1$

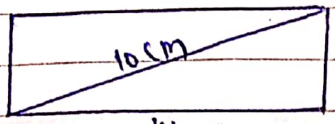
c)

x	-4	-3	-2	-1	0	1	2
x^2	16	9	4	1	0	1	4
$2x$	-8	-6	-4	-2	0	2	4
1	1	1	1	1	1	1	1
$y =$	9	4	1	0	1	4	9
	(1 mark)	(1 mark)	(1 mark)		(1 mark)	(1 mark)	(1 mark)
	$(-4, 9)$	$(-3, 4)$	$(-2, 1)$	$(-1, 0)$	$(0, 1)$	$(1, 4)$	$(2, 9)$

d) NB: on square paper

e) ~~Graph on next page~~

18.a)



$\frac{P}{2} = \frac{28}{2} = 14$

$x = x$ (1mark)

$14 = 14 - x$

$(14-x)^2 + x^2 = 10^2$ (1mark)

$196 - 28x + x^2 + x^2 = 100$ (1 mark)

$2x^2 - 28x + 196 - 100 = 0$

$2x^2 - 28x + 96 = 0$ (1 mark)

$2(x^2 - 14x + 48) = 0$

$\Delta = 196 - 4.1.48$ (1 mark)

$= 196 - 192$

$= 14 = \pm 2$ (1 mark)

$x_1 = \frac{14+2}{2.1} = \frac{16}{2} = 8$ (1 mark)

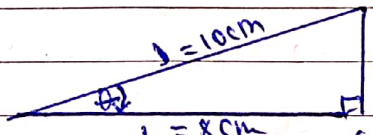
$x_2 = \frac{14-2}{2.1} = \frac{12}{2} = 6$

(1 mark)

$L = 8$ cm

$W = 6$ cm

Area $= L \times W = 8 \times 6 = 48 cm^2$



(1mark) (1mark)

Use of $\tan\theta = \frac{\text{opposite}}{\text{Adjecent}} = \frac{6}{8} = 38.6°$

or $\sin\theta = \frac{opp}{Hy} = \frac{6}{10} = 38.6°$

or $\cos\theta = \frac{Adj}{Hy} = \frac{8}{10} = 38.6°$

18b) $D = 28$

$r = \frac{28}{2} = 14cm$ (1mark)

$A = 4\pi r^2 = 4 \cdot \frac{22}{7} \cdot 14^2$ (1mark)

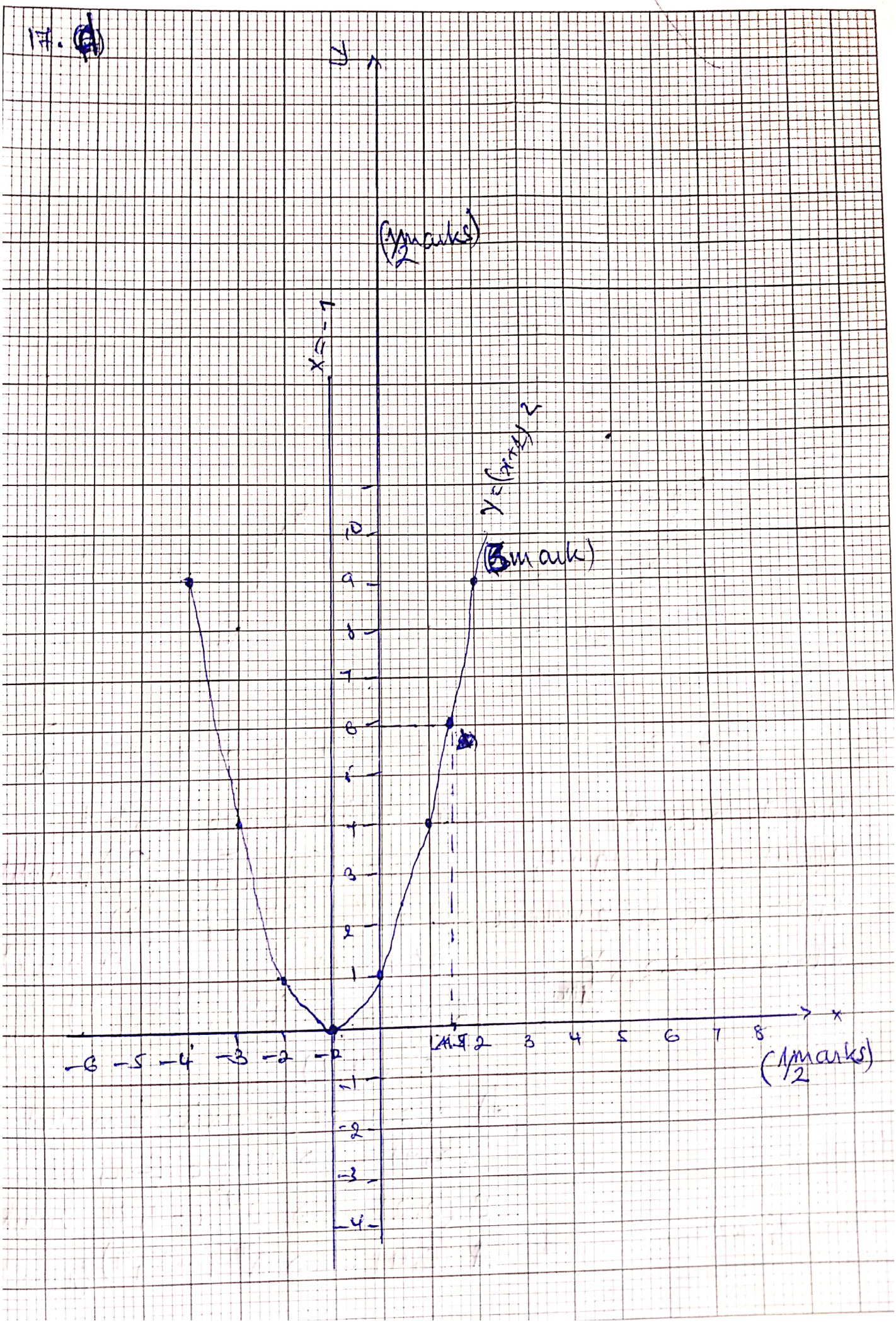
$A = 2464 cm^2$ (1mark)

$V = \frac{4}{3} \pi r^3$

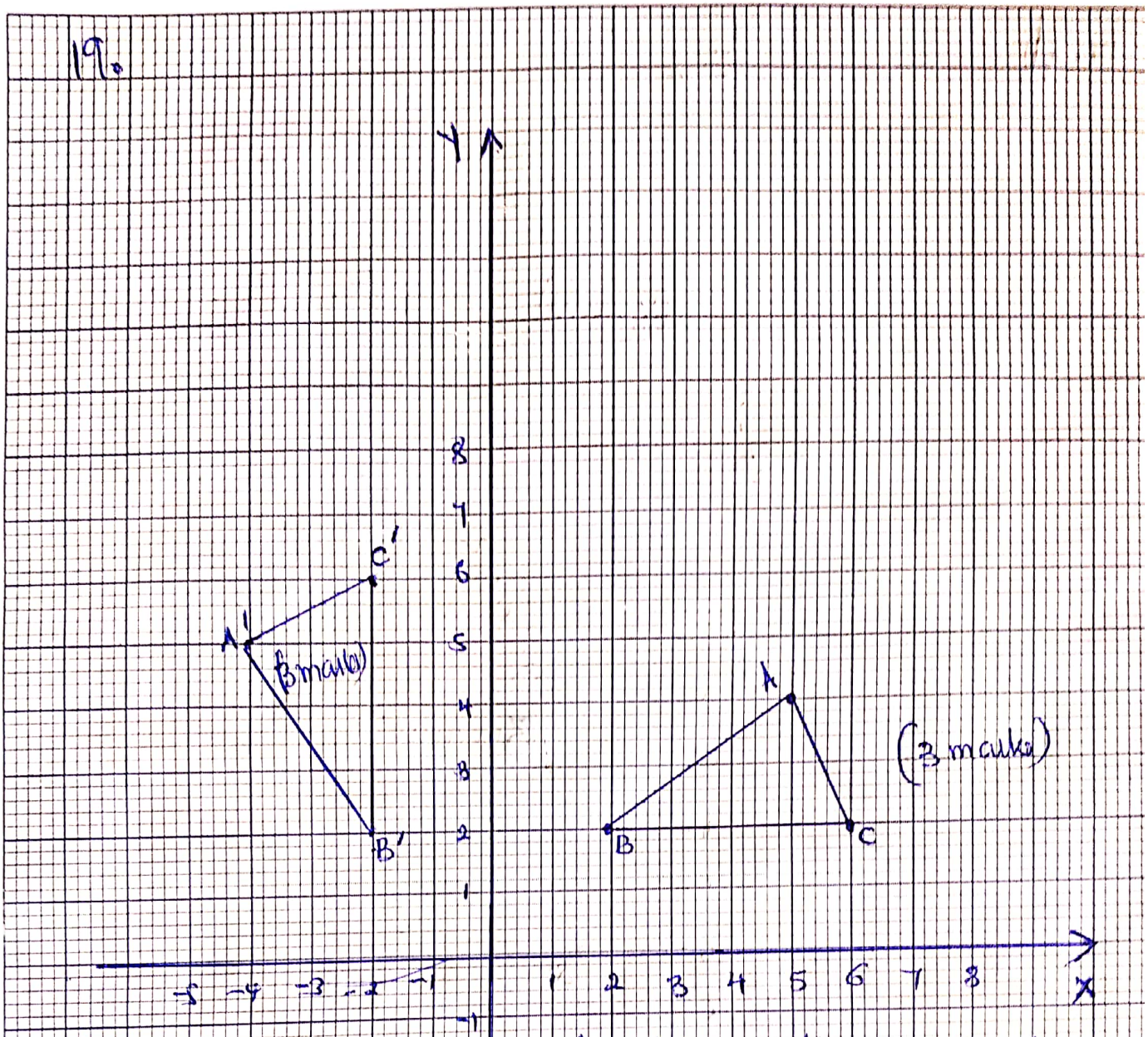
$= \frac{4}{3} \cdot \frac{22}{7} \cdot 14^3$ (1mark)

$V = 11498 cm^3$ (1mark)

17. (a)


y
x
(1/2 marks)
y = (x+4)²
(3 marks)
10
9
8
7
6
5
4
3
2
1
-1
-2
-3
-4
-6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7 8
(1/2 marks)

19.



b) $(x, y) \longrightarrow (-y, x)$

$A(5,4) \longrightarrow (-4,5)$ (1 mark)

$B(2,2) \longrightarrow (-2,2)$ (1 mark)

$C(6,2) \longrightarrow (-2,6)$ (1 mark)

c) translation = Image − object

$t = \begin{pmatrix} 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$ (1 mark)

$B(2,2) \Rightarrow B'' = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (1 mark)

$B''(0,1)$

$C(6,2) \Rightarrow C'' = \begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ (1 mark)

$C''(4,1)$

d) $x = 0$ or y-axis (1 mark)

$(x, y) \longrightarrow$ Image $(-x, y)$

$B(2,2) \longrightarrow B'''(-2,2)$ (1 mark)

$A(5,4) \longrightarrow A'''(-5,4)$ (1 mark)

20. a)

x	f	fx	cf
15	1	15	1
16	8	128	9
17	9	153	18
18	5	90	23
19	0	0	23
20	7	140	30
	$\Sigma f = 30$	$\Sigma fx = 526$	

any box = 0.5 marks
(24 x 0.5 = 12 marks)

b. (i) Modal age = 17. (½ mark)

(ii) Median = Position $= \frac{\frac{n}{2} + \frac{n}{2}+1}{2} = \frac{\frac{30}{2} + \frac{30}{2}+1}{2}$

$= \frac{15^{th} + 16^{th}}{2}$

$= \frac{18+18}{2} = \frac{36}{2} = 18$. (1 mark)

(iii) Range = highest - lowest = 20 - 15 = 5 (½ mark)

(iv) Mean $= \frac{526}{30} = 17.5$ (1 mark)